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Comment on the Matrix Element of O_{11}

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Abstract

The gluon dipole operator, O_{11} , has received much attention recently because it can have a large coefficient in some SUSY extensions of the standard model. We find that the commonly used matrix element of O_{11} of Bertolini, Eeg and Fabbrichesi is in rough (but accidental) numerical agreement with an estimate based on dimensional analysis.

One of the operators in the low energy effective Hamiltonian responsible for $|\Delta S| = 1$ weak decays is the gluon dipole operator, which can be written as ($H_{eff} = C_{11}O_{11}$),

$$H_{eff} = C_{11} \frac{g_s}{8\pi^2} \bar{s}[m_d R + m_s L] T^a G_a^{\mu\nu} \sigma_{\mu\nu} + \text{h.c.}, \quad (1)$$

where $G_a^{\mu\nu}$ is the gluon field strength tensor, and $L, R \equiv (1 \mp \gamma_5)/2$. In the standard model, the coefficient C_{11} is sufficiently small to make the effects of this operator negligible. Beyond the standard model, however, this operator can have a large coefficient [1] and it becomes important to estimate its matrix element. The operator has received much attention recently, in connection with ϵ'/ϵ [2].

The importance of this operator for kaon decays, and in particular for the analysis of ϵ'/ϵ dates back to the Weinberg model of CP violation in the early 80's. It was shown back then, that one had to pay particular attention to the chiral properties of the operator [3] in order to obtain physical amplitudes that obeyed the FKW theorem [4].

The gluon-dipole operator transforms as $(\bar{3}, 3)$ or $(3, \bar{3})$ under chiral rotations and, in the standard model, it is proportional to the light quark masses. It is well known that there are no operators with these transformation properties in the lowest order, $\mathcal{O}(p^2)$, weak chiral Lagrangian [5, 6]. At next to leading order, $\mathcal{O}(p^4)$, there are several operators with the desired properties.

The strong interactions of pions and kaons are described to order $\mathcal{O}(p^4)$ in chiral perturbation theory by the Lagrangian of Gasser and Leutwyler [7]. The ingredients to construct this Lagrangian are the non-linear function $\Sigma = \exp(2i\phi/f)$ which contains the octet of pseudo-Goldstone bosons ϕ and that transforms as $\Sigma \rightarrow R\Sigma L^\dagger$ under $SU_L(3) \times SU_R(3)$. Interactions that respect chiral symmetry are constructed in terms of derivatives of Σ . Explicit chiral symmetry breaking due to the non-zero light quark mass matrix $M = \text{diag}(m_u, m_d, m_s)$ is introduced through the factors S and P ,

$$\begin{aligned} S &= \chi^\dagger \Sigma + \Sigma^\dagger \chi \\ P &= i(\chi^\dagger \Sigma - \Sigma^\dagger \chi) \end{aligned} \quad (2)$$

For our present purpose it suffices to take $\chi = 2B_0 M$, where the parameter B_0 is proportional to the quark condensate $\langle \bar{q}q \rangle$, and relates the current quark masses to the meson masses:

$$\langle \bar{q}q \rangle = -f_\pi^2 B_0, \quad m_K^2 = B_0(m_s + m_d), \quad m_\pi^2 = B_0(m_u + m_d) \quad (3)$$

The leading order weak chiral Lagrangian transforming as $(8_L, 1_R)$ under chiral symmetry was first written down by Cronin [8] and has only one term (We use the notation $L_\mu = i\Sigma^\dagger D_\mu \Sigma$),

$$\mathcal{L}_W^{(2)} = c_2 \text{Tr} \left(\lambda_6 L^2 \right) \quad (4)$$

To introduce explicit chiral symmetry breaking due to the non-zero light quark mass matrix M into the chiral Lagrangian one pretends that the mass matrix transforms

as $M \rightarrow RML^\dagger$ and constructs operators with the desired transformation properties. For the dominant $(8_L, 1_R)$ weak operators it is well known that there is no mass term at order p^2 , in accordance with the FKW theorem.

The operators that occur at next to leading order in the weak chiral Lagrangian have been written down in Ref. [6]. From their general list, those that could correspond to chiral realizations of the operator O_{11} are the ones that contain one factor of the quark masses, they are (only five are independent) [6]

$$\begin{aligned} \mathcal{L}_W^{(4)} = & E_{10} \text{Tr}(\lambda_6 \{S, L^2\}) + E_{11} \text{Tr}(\lambda_6 L_\mu S L^\mu) + E_{12} \text{Tr}(\lambda_6 L_\mu) \text{Tr}(\{L^\mu, S\}) \\ & + E_{13} \text{Tr}(\lambda_6 S) \text{Tr}(L^2) + E_{14} \text{Tr}(\lambda_6 L^2) \text{Tr}(S) + E_{15} \text{Tr}(\lambda_6 i[P, L^2]) \end{aligned} \quad (5)$$

From the point of view of the transformation properties of O_{11} under chiral symmetry all these terms are equally valid. It has been argued in Ref. [11] that the precise form of the short distance operator in Eq. 1 requires a chiral representation in which λ_6 and the quark mass matrix appear next to each other. This requirement reduces the possible operators in Eq. 5 to the ones multiplying E_{13} and the combination $E_{10} - E_{15}$.

Using Eq. 5 we find for the matrix element of O_{11} ,

$$\begin{aligned} C_{11} < \pi^+ \pi^- | O_{11} | K^0 > = & -\frac{2\sqrt{2}}{f_\pi^3} (m_K^2 - m_\pi^2) \left((m_K^2 + 2m_\pi^2) E_{10} + m_\pi^2 E_{11} \right. \\ & \left. + (4m_\pi^2 - 2m_K^2) E_{13} + (m_\pi^2 + 2m_K^2) E_{14} + m_K^2 E_{15} \right) \end{aligned} \quad (6)$$

From this expression it is clear that the matrix element, in general, does not vanish in the limit $m_\pi \rightarrow 0$.

Although the framework of Eq. 5 is completely general, it does not tell us the size of the coefficients E_i . We can estimate the size of the E_{10} , for example, that is needed to match the operator O_{11} by using naive dimensional analysis [9]. For this purpose we write down a Lagrangian with the minimal number of fields that is contained in the term proportional to E_{10} (other terms are related to this one by soft pion theorems), $\mathcal{L} = g_M m_K^2 \partial_\mu K \partial^\mu \pi$ and match g_M to C_{11} following Weinberg [9], with the result

$$\left(E_{10} \right)_{NDA} \sim \frac{f_\pi^2}{4\sqrt{2}} \frac{C_{11}}{8\pi^2} \frac{g_s}{4\pi} \quad (7)$$

From this we write

$$\left(C_{11} < \pi^+ \pi^- | O_{11} | K^0 > \right)_{NDA} \sim (m_K^2 - m_\pi^2) \frac{m_K^2}{f_\pi} \frac{C_{11}}{16\pi^2} \frac{g_s}{4\pi} \quad (8)$$

To evaluate this expression one would use a value of $g_s \sim \sqrt{4\pi}$. Note that an equivalent expression is obtained if one uses the term proportional to E_{13} in Eq. 6 instead.

Bag Model Estimate

An explicit calculation of $\langle \pi | O_{11} | K \rangle$ within the MIT bag model [10], supplemented with a soft pion theorem, led to the estimate of Donoghue and Holstein [3],

$$\left(C_{11} \langle \pi^+ \pi^- | O_{11} | K^0 \rangle \right)_{MIT} = -C_{11} \frac{g_s m_s}{32\pi^2} \frac{A_{K\pi}}{2f_\pi} \frac{m_K^2}{\Lambda^2} \quad (9)$$

Eq. 9, is a trivial rescaling of the actual calculation in Ref. [3]. The factor $A_{K\pi}$ is obtained numerically from the Bag model, $A_{K\pi} = 0.4 \text{ GeV}^3$, and the last factor in Eq. 9 introduces the suppression required by the FKW theorem. In accordance with power counting they choose $\Lambda \sim 1 \text{ GeV}$, corresponding to a matching of O_{11} into an $\mathcal{O}(p^4)$ chiral Lagrangian such as the term that multiplies E_{10} in Eq. 5. Numerically, we find that

$$\left(C_{11} \langle \pi^+ \pi^- | O_{11} | K^0 \rangle \right)_{MIT} \approx 0.8 \left(C_{11} \langle \pi^+ \pi^- | O_{11} | K^0 \rangle \right)_{NDA} \quad (10)$$

Chiral Quark Model Estimate

More recently, Bertolini, Eeg and Fabbrihesi have used a chiral quark model supplemented with some matching conditions to estimate that [11],

$$\left(C_{11} \langle \pi^+ \pi^- | O_{11} | K^0 \rangle \right)_{CQM} = \frac{\sqrt{2}}{f_\pi^3} (m_s - m_d) m_\pi^2 \frac{C_{11}}{16\pi^2} \left(-\frac{11}{4} \langle \bar{q}q \rangle_G \right) \quad (11)$$

There are two points that we want to stress about this expression. First, the last factor in Eq. 11 is the model dependent quantity that arises from their chiral quark model, and that we take at face value. Second, the factor of m_π^2 arises from the requirement that the short distance operator O_{11} match into an order p^4 weak chiral Lagrangian of the form [11]

$$\mathcal{L} \sim \text{Tr} \left[(\Sigma^\dagger M \lambda_6 + \lambda_6 M \Sigma) D^\mu \Sigma^\dagger D_\mu \Sigma \right] \quad (12)$$

Comparing this with the general form, Eq. 5, we see that the only term that is retained is that proportional to the operator whose coefficient is $E_{10} - E_{15}$. As mentioned above, the requirement that the quark mass matrix M and λ_6 appear next to each other selects this term plus the one whose coefficient is E_{13} . This latter one is dropped in Ref. [11] because all products of two traces are suppressed in their model.

The specific numerical result of Ref. [11], however, is very similar to our dimensional analysis estimate. This happens because the m_π^2/m_K^2 suppression that occurs in the weak chiral Lagrangian operators that occur in their matching is compensated by the large numerical coefficient, 11, in Eq. 11. Numerically,

$$\left(C_{11} \langle \pi^+ \pi^- | O_{11} | K^0 \rangle \right)_{CQM} \approx 1.4 \left(C_{11} \langle \pi^+ \pi^- | O_{11} | K^0 \rangle \right)_{NDA} \quad (13)$$

We see that all three estimates are numerically very similar and in agreement with each other within the uncertainty of each approach. However, it is clear that the numerical agreement with the chiral quark model result of Ref. [11] is accidental.

Beyond the Standard Model

In all the cases that we have discussed, we have considered the matrix element of the operator written as in Eq. 1. In this form it appears that the operator vanishes in the chiral limit being proportional to the light-quark masses. This was, in fact, an important ingredient in the matching to a corresponding chiral Lagrangian. However, in some models of interest, this is just an artifact of the normalization; the coefficient C_{11} goes as m_s^{-1} . In this case we want to construct a low energy meson Lagrangian that transforms as $(\bar{3}, 3)$ or $(3, \bar{3})$ but that is not proportional to the light quark masses.

It is possible to write a term without derivatives,

$$\mathcal{L} = \text{Tr}(\Sigma^\dagger h + h^\dagger \Sigma) \quad (14)$$

that does not contribute to $K \rightarrow \pi\pi$ amplitudes once tadpoles are properly subtracted. The leading order amplitude arises from a Lagrangian with two derivatives. In terms of the matrix h whose only non-zero entry is $h_{23} = 1$ we can write, for example,

$$\begin{aligned} \mathcal{L} &= g_{N1} \text{Tr}(h D_\mu \Sigma^\dagger D^\mu \Sigma \Sigma^\dagger) + \text{h.c.} \\ &+ g_{N2} \text{Tr}(\Sigma^\dagger h + h^\dagger \Sigma) \text{Tr}(D_\mu \Sigma^\dagger D^\mu \Sigma) \end{aligned} \quad (15)$$

The first term in Eq. 15 yields a matrix element proportional to m_π^2 , but the second term does not. We find,

$$C_{11} < \pi^+ \pi^- | O_{11} | K^0 > = g_{N2} \frac{2\sqrt{2}}{f_\pi^3} m_K^2 + \mathcal{O}(m_\pi^2/m_K^2) \quad (16)$$

Although we cannot compute g_{N2} , we can again estimate it with naive dimensional analysis following Weinberg. Noting that the operator does not contain terms with only two fields, we find

$$g_{N2} \sim f_\pi^3 \left(\frac{g_s m_s C_{11}}{16\pi^2} \right) \quad (17)$$

resulting in

$$\left(C_{11} < \pi^+ \pi^- | O_{11} | K^0 > \right) \sim m_s m_K^2 \frac{g_s C_{11}}{8\pi^2} \quad (18)$$

This result is equivalent to Eq. 8, differing only by factors of order one that cannot be accounted for with naive dimensional analysis.

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